Numerical VaR Estimation: Realized Moments and the Sinh-Arcsinh Distribution

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ABSTRACT

The shape of profit and loss distribution of an investment is constantly evolving due to the changes in financial fundaments, noise, and shocks. Most value-at-risk models tend to utilize only up to the second moment of the returns distribution. This paper introduces the use of the sinh-arcsinh distribution with time-varying parameters that are numerically estimated from the first four realized moments. Upon applying to the Philippine Stock Exchange Index returns series, the proposed methodology is less conservative but more efficient in capital allocation than the benchmark model TARCH-QMLE. Overall, the results show that the proposed methodology is promising in estimating market risk.

Keywords: realized moments, sinh-arcsinh distribution, VaR, expected shortfall.

1. Introduction

Value-at-Risk (VaR) is one of the popular measures of risk in the financial industry. The VaR is the α^{th} quantile of the returns distribution. It is interpreted as the minimum value of the portfolio that is at risk given a maximum probability of α for a specified time horizon. Let F_r be the distribution of the returns series. The α% VaR is the value l such that the probability that the return r is lower than l must be no larger than α, where $(1-\alpha)$ is the confidence level.

The VaR is formally defined as:

$$VaR_\alpha = \inf\{l \in \mathbb{R} : P(r < l) \leq \alpha\} \quad \textbf{(1)}$$

The usual values for α are 5% or 1% probabilities. Typical time horizons are one-day or ten-day VaR.

Another popular measure of risk is the expected shortfall (ES). The ES is the average value of losses once the return breaches the VaR. The ES is formally defined as:

$$ES_\alpha = \frac{1}{\alpha}\int_0^\alpha q_u(F_r)du = E(r|r \leq VaR_\alpha) \quad \textbf{(2)}$$

where $q_u(F_r)$ is the quantile function of F_r. The ES is affected by the shape of the distribution at the tails. That is, a negatively-skewed leptokurtic distribution may have a higher ES than the Gaussian even if the two distributions have the same VaR.

The usual distributional assumptions in measuring the VaR and ES are the Gaussian distribution and the leptokurtic Student's t distribution. This assumption implicitly implies that at any given time period, the shape of the distribution is fixed with respect to its skewness and kurtosis, which is not the case in practice. In finance, a bull market implies higher occurrences of positive price changes and gains, while the reverse is observed during a bear market. In crisis periods, high volatility is coupled with high probability of extreme losses and low probability of gains. Therefore, a symmetric distributional assumption does not suffice in describing financial data.

The literature had been delving into the use of higher moments in modeling risk; these studies are summarized in Section 2 of this paper. This paper contributes to the literature by tweaking the shape of a flexible distribution, called the Sinh-Arcsinh Distribution, using the realized moments of the data. To forecast the risk measures, such as the VaR and ES, the local linear model is used, as discussed in Section 3. This paper uses the Philippine Stock Exchange Index (PSEi) in implementing the procedure. The PSEi series is divided for model estimation (insample) from 1985 to 2006, and model forecast performance verification (outsample) from 2007 to 2013.

2. Realized Moments, VaR, and VAR

2.1. Realized Moments

The realized moments are a function of actual returns falling within an observation window or interval. For a time-interval t, denote the daily closing price, in logarithms, for intra-period i as $p_{t,i}$. The intra-period log-returns, $r_{t,i}$, is the difference between two succeeding log prices, given as $r_{t,i} = p_{t,i} - p_{t,i-1}$; log returns are common in the literature because of its mathematical tractability. Adapting the intra-period realized volatility of Andersen, Bollerslev, Diebold, & Labys, (2003) (henceforth ABDL), the realized variance for week t, $RVar_t$, is the sum of the intra-week daily returns, given by:

$$RVar_t = \sum_{i=1}^{N} r_{t,i}^2 \quad \textbf{(3)}$$

where $N = 5$, for a 5-day trading schedule. The realized volatility is defined as the square-root of the realized variance scaled by the number of intra-time periods:

$$RV_t = \sqrt{\frac{1}{N} RVar_t} \quad \textbf{(4)}$$

Amaya, Christoffersen, Jacobs, & Vasquez (2015) (henceforth ACJV) computed the realized skewness and kurtosis from the sum of the third and fourth power, respectively, of the intra-period returns, scaled by the realized variances. Following ACJV, the realized skewness, RS_t, and kurtosis, RK_t, are computed by:

$$RS_t = \frac{\sum_{i=1}^{N} r_{t,i}^3}{RVar_t^{2/3}} \tag{5}$$

$$RK_t = \frac{N\sum_{i=1}^{N} r_{t,i}^4}{RVar_t^2} \tag{6}$$

ACJV showed that the realized moments converge in mean square to the integrated moments. They also showed that the realized higher moments augment the information measured by the realized volatility; the realized skewness captures the direction of the abrupt change in equity prices, while the realized kurtosis measures the possible magnitude of the change.

2.2. Modeling Realized Moments

ACVJ (2015) analyzed the effects of realized volatility, skewness and kurtosis on weekly equity returns. The authors observed that realized skewness has a negative impact on next week's stock returns, and realized kurtosis has a positive effect on next week's stock return. They noted that realized volatility and stock returns do not have a strong relationship, when weekly returns were regressed by the realized moments following Fama and MacBeth (1973):

$$r_{i,t+1} = \gamma_{0,t} + \gamma_{1,t} RVol_{i,t} + \gamma_{2,t} RSkew_{i,t} + \gamma_{3,t} RKurt_{i,t} + \phi_t' \boldsymbol{Z}_{i,t} + \epsilon_{i,t+1} \tag{7}$$

where $\boldsymbol{Z}_{i,t}$ is a vector of other explanatory variables for the returns.

ABDL (2003) related the realized volatilities of Deutschemark, Yen and Dollar exchange rates using the vector autoregressive models (Sims, 1980), called as VAR-RV. The authors showed that the VAR-RV is superior to other commonly used approaches, such as the J.P. Morgan's 1997 RiskMetrics, VAR models on returns, AR model on realized volatilities, GARCH(1,1) model, etc. Cayton and Mapa (2015) also modeled the realized moments using the Johnson-SU distribution, a distribution that is also parametrized by higher moments.

ABDL (2001) observed that the realized volatilities are non-normal, and has long-memory dynamics. Studies of Chow et. al. (2009) on the realized volatility of index constituent stocks in Hong Kong, and Thomakos & Wang (2003) on the futures market, noted the long-memory dynamics of the realized volatility, despite the returns being serially uncorrelated. With these concerns, the popular approach is to model the fractionally differenced realized volatility using ARFIMA. On a similar note, Goncalves & Meddahi (2011) applied the Box-Cox transformation to the realized volatility to address the skewness of its distribution. They suggested the use of the inverse of the realized volatility, known as the realized precision, to control the coverage probability of 95% level for integrated volatilities, which is crucial in VaR analysis.

The convergence results assumed that the high-frequency returns are error free. However, actual high-frequency returns suffer from microstructure noise (Andreou & Ghysels, 2002; Bai, Russell, & Tiao, 2008). ACJV (2015) showed that the realized volatility is dominated by microstructure noise as the sampling frequency increases, and that the realized volatility and realized kurtosis only have a small and insignificant bias that does not increase with the sampling frequency. It is recommended in the literature to smoothen the realized volatilities to minimize the effects of measurement error. A popular approach is to subject the realized volatilities to a filtering model that factors out the microstructure noise. Barndorff-Nielsen & Neil (2002), & Meddahi (2002) used the stochastic volatility model, while ABDL (2003) recommended the use of a parsimonious ARMA model. Goncalves & Meddahi (2009) addressed the problem differently by bootstrapping the realized volatilities.

2.3. The Sinh-Arcsinh Distribution

To obtain a more accurate measure of the VaR and ES, the return distribution must be flexible for changes in the skewness and kurtosis of the returns series. Jones & Pewsey (2009) developed the sinh-arcsinh transformation of a normally distributed random variable, yielding a distribution having four parameters that controls directly the location, scale, skewness, and kurtosis. The sinh-arcsinh distribution boasts mathematically tractable properties that increase the ease of matching the distribution's moments with the predicted realized moments. Following the notation of the authors, let Z and $X_{\epsilon,\delta}$ follow the standard normal distribution ϕ and the standardized sinh-arcsinh distribution $f_{\epsilon,\delta}$, respectively. The sinh-arcsinh transformation is given by:

$$Z = \sinh\{\delta \sinh^{-1}(X_{\epsilon,\delta}) - \epsilon\} \tag{8}$$

where ϵ and δ are the skewness and kurtosis parameters, respectively. The distribution of $X_{\epsilon,\delta}$ is given by:

$$f_{\epsilon,\delta}(x) = \{2\pi(1+x^2)\}^{-\frac{1}{2}}\delta C_{\epsilon,\delta}(x)\exp\{-S_{\epsilon,\delta}^2(x)/2\} \tag{9}$$

where $C_{\epsilon,\delta}(x) = \left\{1 + S_{\epsilon,\delta}^2(x)\right\}^{1/2}$. The moments of the standardized sinh-arcsinh distribution is $E\left(X_{\epsilon,\delta}^r\right) = \frac{1}{2^r}\sum_{i=0}^{r}\binom{r}{i}(-1)^i\exp\left\{(r-2i)\frac{\epsilon}{\delta}\right\}$, giving us the first four moments:

$$E\left(X_{\epsilon,\delta}\right) = \sinh(\epsilon/\delta)\, P_{1/\delta} \tag{10}$$

$$E\left(X_{\epsilon,\delta}^2\right) = \frac{1}{2}\left\{\cosh(2\epsilon/\delta)P_{2/\delta} - 1\right\} \tag{11}$$

$$E\left(X_{\epsilon,\delta}^3\right) = \frac{1}{4}\left\{\sinh(3\epsilon/\delta)P_{3/\delta} - 3\sinh(\epsilon/\delta)P_{1/\delta}\right\} \tag{12}$$

$$E\left(X_{\epsilon,\delta}^4\right) = \frac{1}{8}\left\{\cosh(4\epsilon/\delta)P_{4/\delta} - 4\cosh(\epsilon/\delta)P_{2/\delta} + 3\right\}, \tag{13}$$

where $P_q = \frac{e^{1/4}}{(8\pi)^{1/2}}\left\{K_{(q+1)/2}(1/4) + K_{(q-1)/2}(1/4)\right\}$ having $K_\alpha(x)$ as the modified Bessel function of the second kind.[1] Note that the first four moments are completely determined by the two parameters ϵ and δ.

2.4. Local Level Model

The local level model (Shephard and Harvey, 1990) was used in this paper to generate the one-step ahead forecasts of the realized moments. The local level model is a univariate model that utilizes only the historical movement of the realized moments. It also treats the realized moments independent of each other as the forecasting of one realized moments does not include information about the movements of the others moments. The local level model for a time-series y_t is given by:

$$y_t = \mu_t + \epsilon_t, \epsilon \sim N(0, \sigma_\epsilon^2) \quad \textbf{(14)}$$

$$\mu_t = \mu_{t-1} + \xi_t \sim N\left(0, \sigma_\xi^2\right) \quad \textbf{(15)}$$

where μ_t is a latent variable, ϵ_t and ξ_t are random fluctuations with zero means and variances σ_ϵ^2 and σ_ξ^2, respectively. When modeling realized moments, the local level model estimates μ_t of each realized moment, which is the level of the realized moment at a particular point in time. The dynamic and stochastic properties of the model enable the forecasting and calculation of prediction intervals. The model is usually represented in state-space form and estimated using Kalman Filter.

2.5. Evaluation of VaR Methods

Cayton and Mapa (2015) provided a summary of various methods in comparing the performance of VaR models. Basel (1996) recommended to determine the number of exceptions, which is the number of times where the actual risk is beyond the VaR. The capital allocation as buffer for market risk is adjusted based on the number of exceptions.

[1] The modified Bessel functions of the first and second kind are:

$I_\alpha(x) = \sum_{m=0}^{\infty} \frac{1}{m!\Gamma(m+\alpha+1)}\left(\frac{x}{2}\right)^{2m+\alpha}$ and $K_\alpha(x) = \frac{\pi}{2}\left\{\frac{I_{-\alpha}(x) - I_\alpha(x)}{\sin(\alpha\pi)}\right\}$, respectively.

If the VaR method incurs a high number of exceptions, then the capital allocation must be scaled upward. Table 1 gives the Basel scaling factors for capital allocation for various VaR exceptions for 250 trading days, and the scaling factors for a 52 and 53 weeks that will be used in this paper.

Table 1. Test of Unconditional Coverage

Number of Exceptions (Days or Weeks)	250 Days		52 Weeks		53 Weeks	
	% of Exceptions	Scaling Factor for the Market Risk Capital	% of Exceptions	Scaling Factor for the Market Risk Capital	% of Exceptions	Scaling Factor for the Market Risk Capital
1	0.40	3	1.923	3	1.887	3
2	0.80	3	3.846	3.6	3.774	3.6
3	1.20	3	5.769	4	5.660	4
4	1.60	3				
5	2.00	3.4				
6	2.40	3.5				
7	2.80	3.65				
8	3.20	3.75				
9	3.60	3.85				
10	4.00	4				

2.5.1. Likelihood Ratio Tests

Several likelihood ratio tests were developed by Christoffersen (1998) based on the number of exceptions. The first is the Unconditional Coverage test, which tests if the proportion of exceptions is equal to the desired risk probability, with test statistic given by:

$$LR_{uc} = 2\log\left(\left((1-\hat{\pi})(1-p)\right)^{T-T_1}\left(\frac{\hat{\pi}}{p}\right)^{T_1}\right) \sim \chi^2_{(1)} \tag{16}$$

where $\hat{\pi} = T_1/T$ is the proportion of VaR exceptions, T_1 is the number of VaR exceptions, and T is the number of data points, for a given validation period. The rejection of the null hypothesis implies that the actual risk coverage is greater than the desired.

The second is the test of independence, which determines if the VaR exceptions are clustering through time. If the VaR exceptions exhibit clustering, then the violations are dependent through time and are affected by volatility clustering. The test statistic is given by:

$$LR_{ind} = 2\log\left(\frac{(1-\hat{\pi}_0)^{T_{00}}\hat{\pi}_0^{T_{01}}(1-\hat{\pi}_1)^{T_{10}}\hat{\pi}_1^{T_{11}}}{(1-\hat{\pi}_0)^{T_{00}+T_{10}}\hat{\pi}_0^{T_{01}+T_{11}}}\right) \sim\chi^2_{(1)} \tag{17}$$

where $\hat{\pi}_i = T_{i1}/(T_{i1}+T_{i0})$, T_{00} is the number of two consecutive periods with no exceptions, T_{01} number of periods with no exceptions followed by an exception, T_{10} number of periods with exceptions followed by no exceptions, and T_{11} number of two consecutive days with exceptions. Rejection of the null hypothesis implies that the VaR method suffers from exception clustering.

2.5.2. VaR Methodology Quality Statistics

The third is the test for conditional coverage, which is a joint test of the unconditional coverage and independence. The test statistic is given by:

$$LR_{cc} = LR_{uc} + LR_{ind} \sim\chi^2_{(2)} \tag{18}$$

Rejection of the null implies that the actual risk is higher than the desired risk.

The other set of evaluation measures is composed of statistics that describe some qualities of the VaR approach. The first quality is conservatism that is measured by the mean relative bias (MRB) by Engel and Gizycki (1999). The MRB statistic is given by:

$$MRB_i = \frac{1}{T}\sum_{t=1}^{T}\frac{VaR_t - \overline{VaR}_t}{\overline{VaR}_t} \tag{19}$$

where $\overline{VaR}_t = \sum_{i=1}^{N} VaR_{it}$, where $i \in \{1,\ldots,N\}$ indicates the i^{th} VaR methodology out of N approaches. The higher the MRB implies that the methodology is more conservative than the other approaches.

The second quality is accuracy which is measured by the average quadratic loss function (AQLF) also proposed by Engel and Gizycki (1999). The AQLF formula is given by:

$$AQLF = \frac{1}{T}\sum_{t=1}^{T} L(VaR_t, r_t) \quad \textbf{(20)}$$

$$L(VaR_t, r_t) = \begin{cases} 1 + (VaR_t - r_t)^2 & if\ VaR_t > r_t \\ 0 & if\ otherwise \end{cases} \quad \textbf{(21)}$$

A lower AQLF implies more accurate VaR methodology in accounting potential losses than the others.

The third is a measure for efficiency, which is the average market risk capital (AMRC) by Basel (1996). The formula of the AMRC is given by:

$$AMRC = \frac{1}{T}\sum_{i=t-1}^{T} \max\left[-\frac{k}{60}\sum_{j=t-1}^{t-60} VaR_j\,, VaR_{t-1}\right] \quad \textbf{(22)}$$

where k is the scaling factor for market risk Capital from Table 1. A low AMRC implies low risk capital needed to be allocated.

3. VaR Using the Modeling Realized Volatility and Sinh-Arcsinh Distribution

This section discusses the procedure in measuring the VaR using the realized moments and the Sinh-Arcsinh distribution.

1. The realized moments are computed from the intra-period returns using the Equations (4), (5) and (5).
2. The local level model is used to forecast the realized moments.
3. The parameters ϵ and δ of the sinh-arcsinh distribution are numerically determined by solving the following equations:

$$\mathrm{S} = \frac{E\left(X_{\epsilon,\delta}^3\right) - 3\mu\sigma^2 - \mu^3}{\sigma^3} \quad \textbf{(23)}$$

$$\mathrm{K} = \frac{E\left(X_{\epsilon,\delta}^{4}\right) - 4\mu\sigma^{3}\mathrm{S} - 6\mu^{2}\sigma^{2} - \mu^{4}}{\sigma^{4}} \quad \textbf{(24)}$$

where μ and σ are the realized mean and volatility of the series, respectively, within a given observation window.

4. The $\alpha\%$ VaR is computed using the quantiles of the normal distribution expressed as $\Phi(Z) = \alpha$, where Φ is the normal distribution function. Substitute the VaR in the sinh-arcsinh transformation given by Equation (8) to get:

$$\Phi(\sinh\{\delta\, sinh^{-1}(VaR_{\alpha}) - \epsilon\}) \ = \alpha \quad \textbf{(25)}$$

Solving for VaR_{α} yields:

$$VaR_{\alpha} = \sinh\left\{\frac{\sinh^{-1}\left(\Phi^{-1}(\alpha)\right) + \epsilon}{\delta}\right\} \quad \textbf{(26)}$$

where Φ^{-1} is the quantile function of the normal distribution.

The data used in this paper is the daily closing value of the PSEi from January 2, 1985 to December 29, 2017, translating to 8,608 daily observations excluding weekends. The realized moments are computed using the daily intra-week data. The data is divided into the training dataset, which spans from 1985 to 2007 for a total of 1,148 weeks, and the validation dataset, which spans from 2007 to 2017 for a total of 574 weeks. All in all, there is a total of 1,722 weeks in the series. The validation period was selected because it captures the 2007 global financial crisis. Conclusions based on the results are applicable only to modeling the weekly PSEi returns.

The VaR approach using the realized moments and the sinh-arcsinh distribution, henceforth RM, will be compared with the threshold autoregressive conditional heteroskedasticity model, henceforth TARCH, as benchmark. The expected shortfall, likelihood ratio tests and the VaR quality statistics will be computed for the two methodologies.

4. Results

4.1. PSEi Stylized Facts

The PSE Composite Index (PSEi) is the main market index of the Philippine Stock Exchange. It is the weighted average price of 30 listed stocks, selected to represent the general movement of the market. The composition of the 30 companies may change depending on the company fundamentals, as assessed by the PSE.

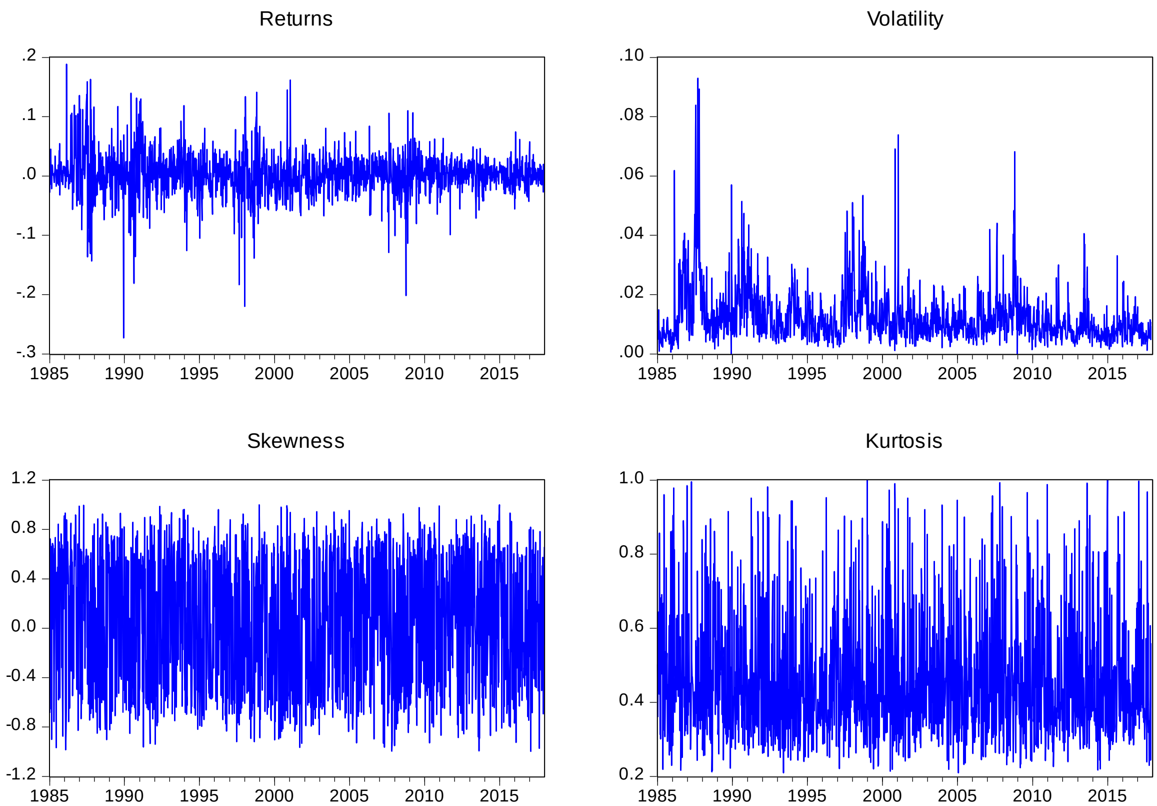
Figure 1. Daily Closing Value of PSEi

Figure 1 displays the daily closing value, in levels and logarithms, of the PSEi; the shaded area corresponds to the validation dataset. It is noteworthy to mention the large downward trend induced by the 1989 Coup Attempt, 1991 Gulf War Crisis, 1997 Asian Financial Crisis, and the 2008 Financial Crisis. Signs of market recovery is evident after the Asian Financial Crisis, but was reversed due to the 2000 DotCom Crisis and

September 11, 2001 attack.[2] The sample also covers the credit rating upgrade of the Philippines to investment grade by Fitch Ratings as announced on March 28, 2013. The shaded region from 2007 to 2017 is the validation period for the VaR approaches.

The intra-week realized moments are computed from the daily PSEi returns, for a total of 1424 datapoints. Figure 2 gives the graph of the realize moments. As expected, the higher realized moments are dominated by market noise. The movement of the realized skewness and kurtosis are indiscernible; nonetheless, the crisis periods register on the realized volatility graph.

Figure 2. Weekly PSEi Realized Moments



The largest single-day decline in the PSEi was on December 11, 1989, where the index suffered week-on-week decrease of around 27%, after stock market trading was halted for more than one week because of the December 1989 Coup Attempt. The maximum week-on-week increase of around 19% is attributed to the February 1986 People Power

[2] Aquino (2004) provides a comprehensive list explaining the shocks or irregularities that affected the PSEi from July 1987 to May 2004.

Revolution ousting President Ferdinand Marcos. On the graph of the realized volatility, the isolated spike on January 22, 2001 is due to the ousting of President Joseph Ejercito Estrada, which was replaced by then Vice President Gloria Macapagal Arroyo. The spike on the realized volatility on October 27, 2008 was due to the Financial Crisis that led to the decline of East Asian stock prices led by large Asian markets of Hong Kong and Japan. The realized skewness and kurtosis exhibit high variance that inhibits the visualization of its movements.

Figure 3. Histogram of PSEi Weekly Realized Moments

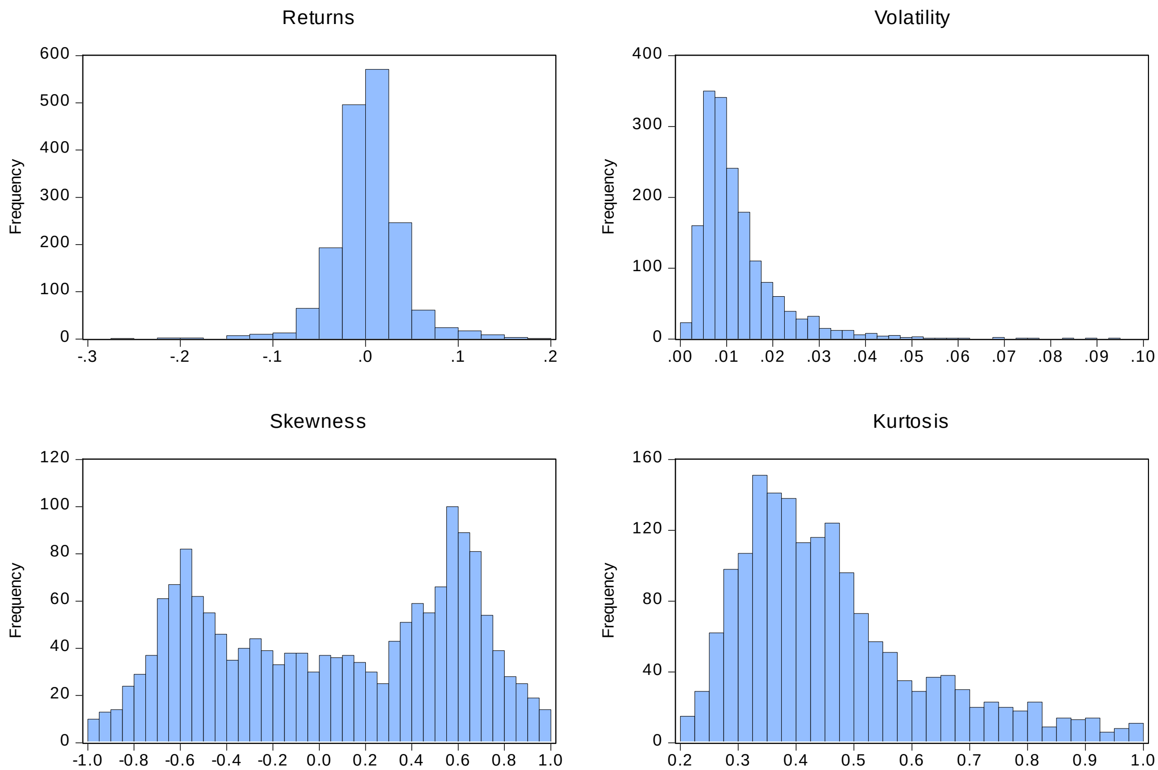
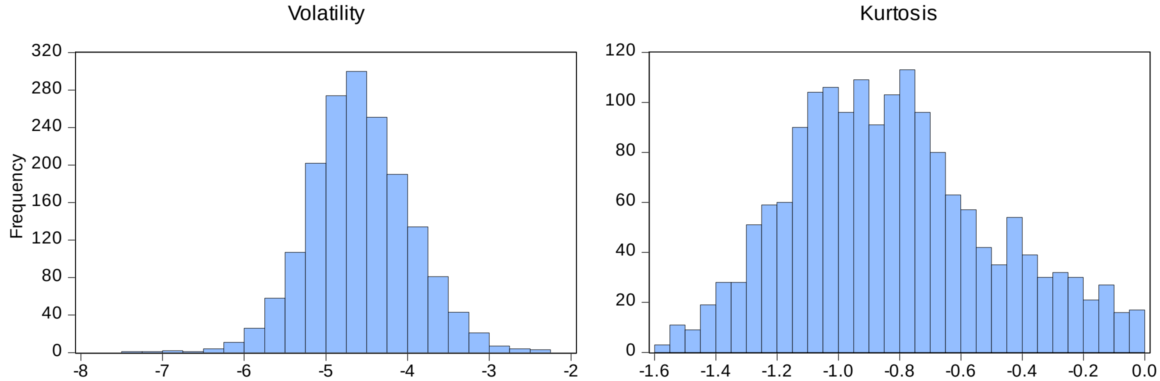


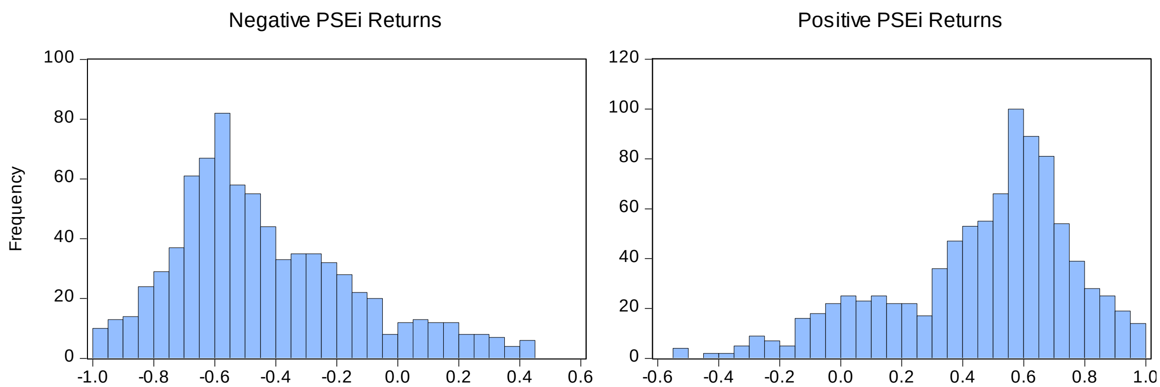
Figure 3 gives the histograms of the realized moments of the PSEi. The distribution of the returns is almost symmetric but with high kurtosis, as expected with financial returns data. The realized volatility & realized kurtosis distributions are highly skewed, while the distribution of the skewness seems to be double-peaked. Getting the logarithms of the two realized moments reduces the skewness of their distributions, as seen in Figure 4. The reduction in skewness would help improve the forecasting performance of the local level model.

Figure 4. Histogram of Realized Volatility and Kurtosis (in Logarithm)



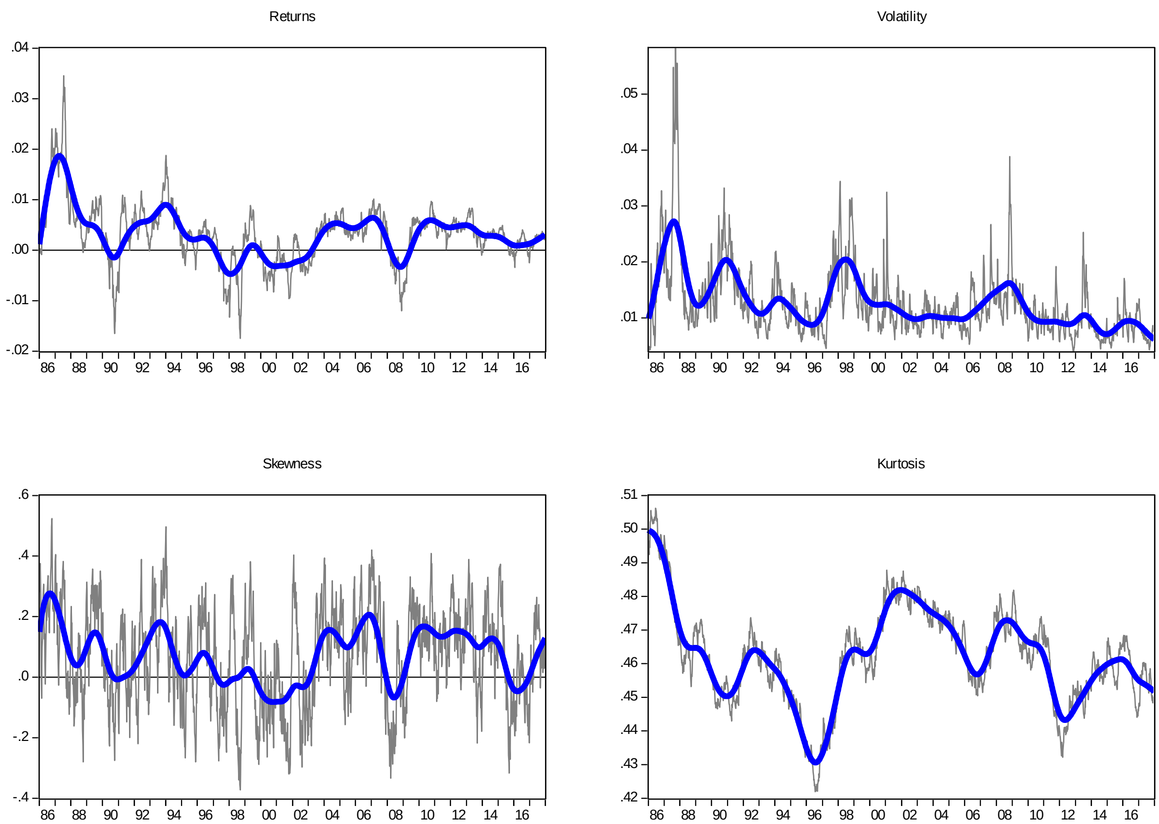
The distribution of the realized skewness seems to have a bimodal distribution. Figure 5 displays the realized volatility distribution for positive and negative PSEi returns. A negative realized skewness is evident for negative PSEi returns; while a positive realized skewness is observed for positive PSEi returns.

Figure 5. Histogram of Realized Skewness of Positive and Negative PSEi Returns



As also observed by other studies that the realized moments are dominated by noise, the realized moments were smoothed to visualize their movements. Smoothing procedure shows the hidden cycles of the realized moments through time as the economy moves from a tranquil to a crisis state. Figure 6 graphs the Holt-Winters Exponential Smoothing and its corresponding Hodrick-Prescott Filter of the PSEi's realized moments. We see from the HP filtered series of the realized skewness that it is on the negative side of zero only during around 1997-1998, 2000-2001, 2008, and 2015-2016, which are crisis periods as defined earlier, except for the 2015-2016 period. There is an increase in the realized volatility and realized kurtosis during 1997-1998 and 2998 crises periods. The negative 2000-2001 negative skewness episode only registered an upward nudge to the kurtosis and not on the volatility. The 2015-2016 period is remarkable as it is the only negative skewness period with no drastic change in the other moments. A combination of a positive skewness and, a low volatility and kurtosis, can be seen during bull market or upward market rally. The reverse can be seen during bear market or downward market rally.

Figure 6. Weekly PSEi Realized Moments: Holt-Winters Exponential Smoothing with HP Filter

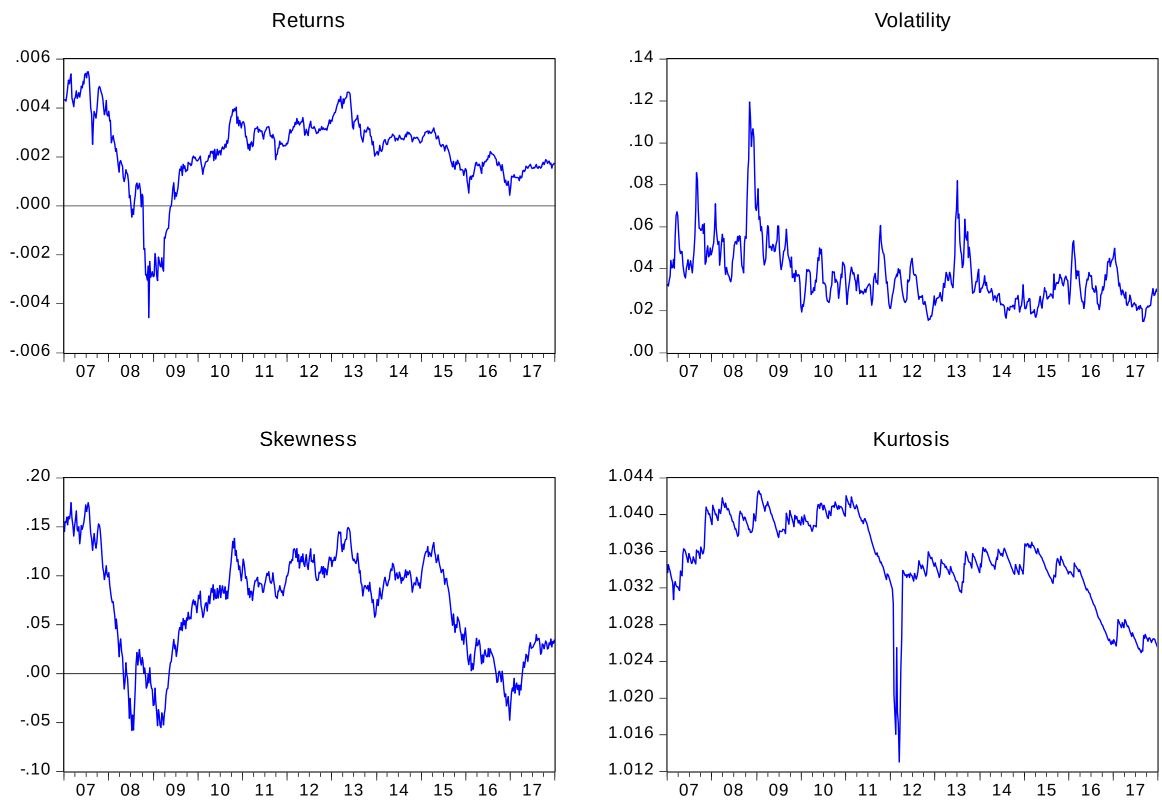


Despite the exponential smoothing producing a good graphical representation of the realized moments' movement, it is not recommended to use the smoothened series as an input to an autoregressive model. The exponential smoothing procedures force the series to be correlated with its previous values, since past values are given exponentially decaying weights.

4.2. Local Level Model Results

The local level model weekly one-step ahead forecasts of the realized moments for the validation period 2007 to 2017 are presented in Figure 7. The upper limit of the 99% prediction interval of the realized volatility and kurtosis were extracted to have a conservative estimate of the VaR. The upward adjustment was also made to consider the highly volatile nature of the realized moments.

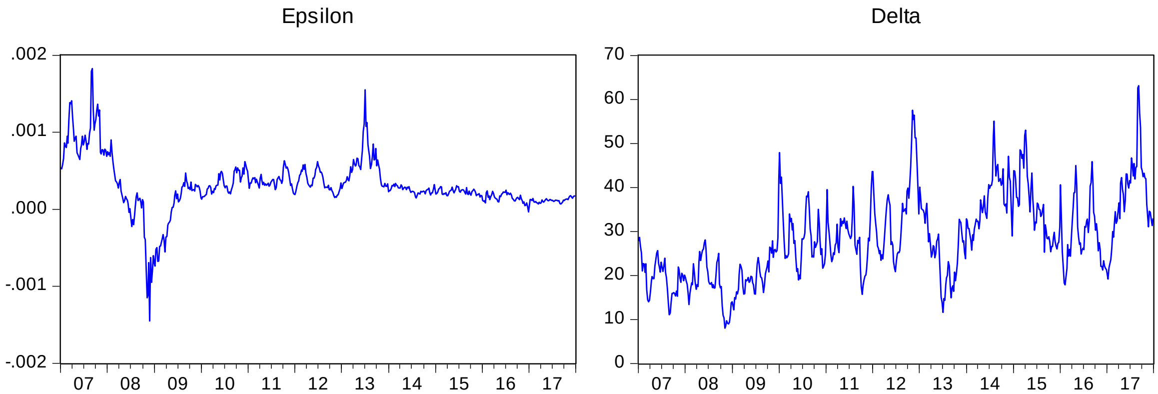
Figure 7. Weekly PSEi Realized Moments: Local Linear Smoothing (2007-2017)



4.3. Numerical Estimation of Sinh-Arcsinh Parameters

The ϵ and δ parameters of the sinh-arcsinh distribution were numerically estimated by solving the system of Equations (23) and (24). Figure 8 gives the movement of the parameters from 2007 to 2017. The numerical estimation of the parameters for all periods is successful in finding the solution to the system of equations, yielding the RHS of the system virtually equal to the LHS. Note that the movement of the parameters has some resemblance to that of the one-step ahead realized moment forecasts, especially on periods with drastic changes. However, the two parameters do not uniquely represent the movement of a single realized moment as the moments are nonlinear functions of the parameters.

Figure 8. Sinh-Arcsinh Estimated Parameters (Weekly; 2007-2017)



4.3 Value-at-Risk Assessment

The one-period ahead 1% VaR was calculated by getting the 1% quantile of the sinh-arcsinh distribution given the estimated parameters for each period. Figure 9 shows the actual weekly PSEi return series and the VaR using the realized moments and sinh-arcsinh distribution, denoted by VaR RM in the graph. It also shows the VaR calculated using a TARCH model estimated using quasi-maximum likelihood estimation (QMLE) as the benchmark model, denoted by VaR TARCH.

The VaR RM is generally higher than the VaR TARCH. This implies the realized moments and the sinh-arcsinh distribution gives lower expected losses than the TARCH-QMLE generally across the validation sample. The lower estimated VaR translates to lower capital allocation for reserves as buffer for volatile investments. This leads to higher liquidity and more capital for investing in higher yielding assets. It can also be observed that the VaR RM is more sensitive to shocks, as a single shock, say at the start of 2013, lead to a sustained higher VaRs in the subsequent period than that of the VaR TARCH. Also, the presence of a substantial shock makes the VaR RM lower than the VaR TARCH. Nonetheless, the VaR RM is also quick to taper-off and increase toward zero if a shock was not sustained. A number of exceptions or VaR violations can be observed during the crisis period of 2007 to 2008 for both approaches.

Figure 9. Returns Series and VaRs (Weekly; 2007-2017)

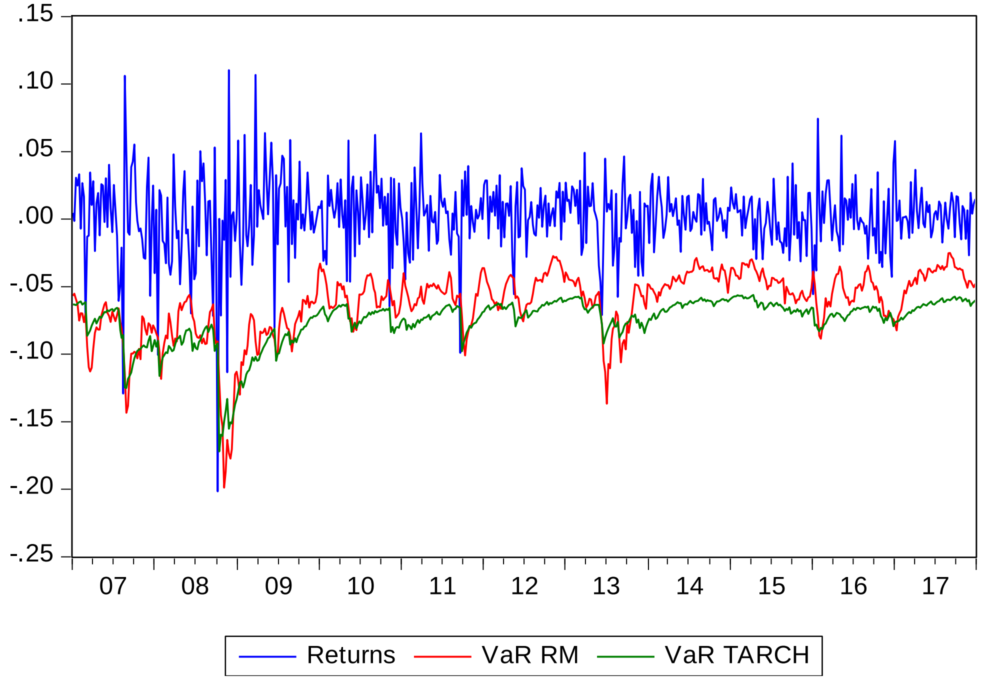


Table 2 lists the summary statistics of the exceptions for the two approaches; the table includes the number of weeks per year, the number of exceptions, and the proportion of exceptions by year. It could be noticed that the VaR RM has generally more exceptions or VaR violations during the crisis period of 2008: 4 exceptions as against 2 under the TARCH. The approach also has exceptions for years 2010, 2012, and 2016 where the TARCH experienced no exceptions. A VaR violation of 1 instance is equivalent to 1.923% given a 52-week year, which is higher than the coverage probability of 1%. In total, the percentages of exceptions are 1.7% and 0.87% for the RM and TARCH approaches, respectively.

Table 2. Summary Statistics of VaR Exeptions

Year	N		Exception		% of Exceptions	
	RM	TARCH	RM	TARCH	RM	TARCH
2007	52	52	2	2	3.846	3.846
2008	53	53	4	2	7.547	3.774
2009	52	52	0	0	0.000	0.000
2010	52	52	1	0	1.923	0.000
2011	52	52	1	1	1.923	1.923
2012	52	52	1	0	1.923	0.000

Year	N		Exception		% of Exceptions	
	RM	TARCH	RM	TARCH	RM	TARCH
2013	53	53	0	0	0.000	0.000
2014	52	52	0	0	0.000	0.000
2015	52	52	0	0	0.000	0.000
2016	52	52	1	0	1.923	0.000
2017	52	52	0	0	0.000	0.000
Total	574	574	10	5	1.742	0.871

Table 3 shows the test of unconditional coverage results by year. Years with no exceptions have undefined LR test statistic, and cannot be tested for unconditional coverage. The testable years under the TARCH have p-values greater than any usual level of significance, implying that the proportions of exceptions in Table 2 are not significantly different from the desired risk probability of 1%. Such is also the case for the RM except for the year 2008 where the percentage of exceptions reached to 7.5%. Overall, the actual proportion of exceptions is not significantly different from the desired across the validation sample.

Table 3. Test of Unconditional Coverage

Year	RM		TARCH	
	LR	p-value	LR	p-value
2007	2.471	0.116	2.471	0.116
2008	9.464	0.002	2.414	0.120
2009	-	-	-	-
2010	0.352	0.553	-	-
2011	0.352	0.553	0.352	0.553
2012	0.352	0.553	-	-
2013	-	-	-	-
2014	-	-	-	-
2015	-	-	-	-
2016	0.352	0.553	-	-
2017	-	-	-	-
Total	2.615	0.106	0.101	0.751

Table 4 shows the performance of the two methodologies in terms of conservatism, accuracy, and efficiency. The MRBs for the RM are the negative MRBs of the TARCH because there are only two models being compared. The RM has negative MRB except for 2007, which gradually reduced in value until 2012. Overall, the lower MRB of the RM implies that it is less conservative than the TARCH.

The AQLF gives the same results as the proportion of exceptions in Table 2. The RM has an AQLF that is twice than that of the TARCH, which implies that the TARCH is more accurate than the RM. However, this statistic is only computed for years 2007, 2008 and 2011 when both the RM and TARCH AQLFs are nonzero. Due to the small sample size, the AQLF may not be able to represent VaR performance accuracy.

Overall, the AMRC of the TARCH is more than that of the RM by about 20%. This implies that the RM has lower allocation of risk capital than the TARCH on average. This is also true across all years in the validation sample, except for the year 2007. Generally, the lower AMRC implies that the use of RM frees-up reserve capital that can be converted into higher yielding investments, as compared to the TARCH.

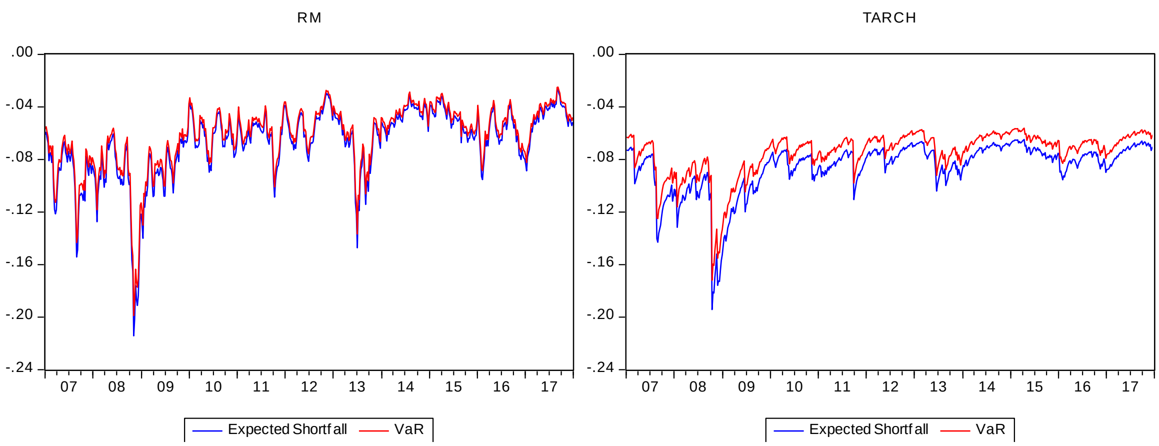
Table 4. Statistical Measures for VaR Comparison

Year	Conservatism (MRB)			Accuracy (AQLF)			Efficiency (AMRC)		
	RM	TARCH	TARCH/RM	RM	TARCH	TARCH/RM	RM	TARCH	TARCH/RM
2007	0.009	-0.009	1.000	0.038	0.039	1.00	0.261	0.250	0.958
2008	-0.049	0.049	1.000	0.076	0.038	0.50	0.265	0.293	1.103
2009	-0.077	0.077	1.000	0.000	0.000	-	0.279	0.308	1.102
2010	-0.116	0.116	1.000	0.019	0.000	0.00	0.170	0.213	1.250
2011	-0.126	0.126	1.000	0.019	0.019	1.00	0.177	0.222	1.257
2012	-0.153	0.153	1.000	0.019	0.000	0.00	0.158	0.203	1.283
2013	-0.046	0.046	1.000	0.000	0.000	-	0.195	0.208	1.069
2014	-0.191	0.191	1.000	0.000	0.000	-	0.137	0.198	1.447
2015	-0.164	0.164	1.000	0.000	0.000	-	0.133	0.187	1.406
2016	-0.123	0.123	1.000	0.019	0.000	0.00	0.166	0.212	1.279
2017	-0.196	0.196	1.000	0.000	0.000	-	0.142	0.197	1.389
Average	-0.112	0.112	1.000	0.017	0.009	0.50	0.188	0.226	1.203

4.4. Expected Shortfall

Figure 9 shows the expected shortfall and the VaR of the two methodologies. It can be observed that the expected shortfall of the RM is closer to the VaR than that of the TARCH. This implies that the expected losses of the RM, given that the VaR is breached, are less than that of the TARCH. This result is achieved because of the flexibility of the sinh-arcsinh distribution to changes in the skewness and kurtosis.

Figure 9. VaR and Expected Shortfall (Weekly; 2007-2017)



5. Conclusion

This paper introduces the use of the sinh-arcsinh distribution evaluated using the realized moments in estimating the VaR of the PSEi. Results show that the proposed methodology is generally better than the benchmark model TARCH-QMLE. The proposed methodology has higher number of VaR exceptions during times of crisis, particularly in 2007 and 2008. There are also several other VaR exceptions that are within the acceptable thresholds. Moreover, the proposed model is less conservative and more efficient in allocating capital than the benchmark. The proposed model also has lower expected shortfall in absolute value than the benchmark model, implying lower expected losses during instances of VaR exceptions.

These conclusions are applicable only to the PSEi returns series. Further studies must be done to assess the performance of the proposed methodology. Nonetheless, the proposed methodology is promising in estimating market risk. Improvements can be done in the forecasting of the realized moments to include dynamic interrelationships, and to distinguish and cancel out the noise from the signal. Moreover, modeling the realized skewness can be improved by incorporating asymmetric effects.

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